

METAL RICH RR LYRAE VARIABLES: II. THE PULSATIONAL SCENARIO

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ABSTRACT

We present a theoretical investigation on the pulsational behavior of metal-rich RR Lyrae variables over the range of evolutionary parameters suitable for stars with metallicities $Z = 0.006, 0.01$ and 0.02 . With the addition of similar results for metal-poor pulsators we discuss the theoretical pulsational scenario covering the metallicity range from $Z = 0.0001$ to 0.02 .

By connecting pulsational constraints to evolutionary prescriptions for He burning stars we discuss the observed behavior of the RR Lyrae population in the Galactic field. We find that the distribution of field *ab*-type RR Lyrae stars in the period-metallicity plane can be easily understood within the framework

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of the present theoretical scenario, suggesting that the Oosterhoff dichotomy also affects field variables.

Theoretical predictions concerning the amplitude-period diagram are discussed and compared with observational data. We find a general agreement for metal-poor ($[\text{Fe}/\text{H}] < -1.4$) RR Lyrae stars, whereas more metal-rich variables show amplitudes smaller than those predicted for pulsators originated from old, low-mass evolving stars. Alternatively, the agreement between theory and observations would require that a substantial fraction of metal-rich RR Lyrae variables in the Galactic field were younger than ≈ 2 Gyr.

The comparison between the pulsational behavior of RR Lyrae either in the Galactic field or in the Galactic bulge discloses the evidence that, at least as far as RR Lyrae variables are concerned, the metal-rich components of the bulge and of the field population appear quite similar. We finally suggest that the peak at $\log P \simeq -0.55$ in the period frequency distribution of first overtone RR Lyrae stars in the Large Magellanic Cloud, considered as possible evidence for second overtone pulsators, could be more simply taken as evidence of a metal-rich stellar population.

Subject headings: Galaxy: stellar content – stars: evolution – stars: horizontal branch – stars: oscillations – stars: variables: RR Lyrae

1. INTRODUCTION

In a previous paper (Bono et al. 1996, hereinafter Paper I) we investigated the theoretical predictions concerning metal-rich, low-mass evolutionary structures undergoing radial pulsational instabilities during their central He burning phase. Summarizing the results, we found that if the increase of He with metals is taken into account and if the stars are old enough (i.e. older than a few billion years), then He burning structures with metallicity Z between 0.01 to 0.02 should cross the region for radial instability at luminosities which range from $\log L/L_{\odot} \simeq 1.52$ to 1.54, with stellar masses from $M \simeq 0.55$ to $0.58 M_{\odot}$. However, in the same Paper I we showed that if younger He burning stars are pushed into the instability region by a more efficient mass loss during the Red Giant Branch (RGB) phase, one could expect much lower luminosities together with much smaller masses. For ages close to 1 Gyr and by assuming a very efficient mass loss ($\approx 1.6 M_{\odot}$) we obtain as lower limits $\log L/L_{\odot} \simeq 1.1$ for the ZAHB luminosity level and $M/M_{\odot} \simeq 0.36$ for the stellar mass.

Taking into account such evolutionary scenario, in this paper we approach the pulsational behavior of He burning models. The aim is to provide a theoretical scenario

to be compared with observational data of metal-rich RR Lyrae stars observed both in the Galactic field and in the Galactic bulge. Since the pulsational behavior depends not only on the structural parameters (stellar mass, luminosity and effective temperature) but, mainly at larger metallicities, also on the amount of metals (see Bono, Incerpi & Marconi 1996, hereinafter BIM), several *ad hoc* sequences of nonlinear, nonlocal and time-dependent convective models of RR Lyrae variables have been computed and discussed.

In the next section we investigate the region of the HR diagram where evolutionary stellar structures with solar metallicity ($Z=0.02$) reveal to be unstable against radial pulsation, reporting the full results of nonlinear computations. Section 3 gives similar results but for the cases $Z=0.01$ and $Z=0.006$. In Section 4 theoretical constraints are compared with observational data of RR Lyrae stars in the Galactic field, also with reference to previous results concerning metal-poor RR Lyrae stars, while Section 5 deals with variables in the Galactic bulge. The summary of the leading results of this investigation and some final remarks on RR Lyrae stars in the MACHO database close the paper.

2. SOLAR METALLICITY RR LYRAE VARIABLES

As a first step in approaching the pulsational scenario concerning metal-rich pulsators we investigate the location in the HR diagram of the regions where stellar envelope models become pulsationally unstable. Following the evolutionary results, this investigation has been performed by assuming fixed stellar mass ($M/M_{\odot}=0.53$) and chemical composition ($Z=0.02$, $Y=0.28$). The HR diagram has been explored with steps of 100 K in the effective temperatures for six choices of stellar luminosity, namely $\log L/L_{\odot} = 1.21, 1.41, 1.51, 1.61, 1.81$ and 2.00 . As shown by the evolutionary results given in Paper I, this choice allows for a full coverage of the pulsational candidates of He burning structures with ages in the range $1 \div 20$ Gyr and takes into account both the ZAHB and the off-ZAHB evolutionary phases. The nonlinear theoretical approach adopted for this analysis has been already discussed in a previous series of papers (e.g. see Bono & Stellingwerf 1994, hereinafter BS; Bono et al. 1996, hereinafter BCCM) and therefore it has not been discussed further.

Figure 1 reports the result of this investigation (solid lines), disclosing the regions in the HR diagram where the fundamental and/or the first overtone mode result unstable. Similar results but for metal-poor stars, as given by BCCM, are also shown (dashed lines). The dotted lines give the zero age horizontal branch (ZAHB) luminosity at $\log T_e=3.85$ for "old" structures (see Paper I) and for the labeled assumptions about metallicity. Around this luminosity level, and for each assumed mass, moving from higher to lower effective temperatures the different lines show the first overtone blue edge (FOBE), the fundamental

TABLE 1
Fundamental and first overtone boundaries.

$\log L/L_{\odot}$	FOBE ^a (K)	FBE ^b (K)	FORE ^c (K)	FRE ^d (K)
$M = 0.53M_{\odot}, Y = 0.28, Z = 0.02$				
2.00		6650		4950
1.81	6750	6750	6650	5350
1.61	7050	6950	6150	5650
1.51	7150	6850	6725	5850
1.41	7350	6950	6550	5850
1.21	7550	6850	6650	6150
$M = 0.58M_{\odot}, Y = 0.255, Z = 0.01$				
1.65	7050	6950	6550	5650
1.57	7150	7050	6550	5750
1.51	7150	7050	6550	5850
$M = 0.58M_{\odot}, Y = 0.255, Z = 0.006$				
1.55	7250	7150	6650	5850

^aEffective temperature of first overtone blue edge.

^bEffective temperature of fundamental blue edge.

^cEffective temperature of first overtone red edge.

^dEffective temperature of fundamental red edge.

blue edge (FBE), the first overtone red edge (FORE), and the fundamental red edge (FRE). Luminosities and effective temperatures of the computed pulsational boundaries are listed in Table 1.

Figure 1 shows that moving from extreme population II to "solar" pulsators the luminosity of the stars and the topology of the instability strip move in such a way that the range of effective temperatures where stars experience pulsational instability appears largely unchanged. As a matter of fact, mainly for the effect of the decreased mass, the solar metallicity strip largely reproduces the topology of the metal-poor case, but at smaller luminosities. According to such a topological transformation, in the solar case we find that the upper luminosity level which allows for the occurrence of first overtone pulsators has now decreased down to $\log L \simeq 1.8$. Above this limit only fundamental pulsators are expected.

Selected pulsating models for each assumed level of luminosity and for given values of the stellar effective temperature have been followed in time until the radial motion approaches its asymptotic amplitude. Tables 2 and 3 in the Appendix give selected quantities for all the computed models while Fig. 2 and Fig. 3 show an atlas of the bolometric light curves for both fundamental and first overtone pulsating modes. Comparison with the light curves of metal-poor pulsators allows for some general comments. As far as the fundamental mode is concerned, the overall light curve morphology found in the metal-poor case appears translated to lower luminosity levels, including the occurrence of secondary features like "Bumps" and "Dips" through the pulsation cycle. However, for $\log L/L_{\odot} = 1.81$ the two bluest models of the metal-rich series lack the "Dip" on the rising branch and present a little "Bump" on the decreasing branch, a feature which is not found in any of the metal-poor models. Concerning both the instability strip topology and the light curve morphology as a whole it appears that metal-rich pulsators largely reproduce, although at lower luminosities, the theoretical scenario presented in BS and in BCCM for metal-poor RR Lyrae variables, with some minor variations which could be revealed only in very accurate photometric light curves or through a Fourier analysis of the theoretical light curves.

First overtone light curves again display properties similar to those of metal-poor pulsators, but shifted toward lower luminosity levels if compared to globular cluster pulsators. Again we find that at the lower luminosities, i.e. below the predicted HB luminosity level, first overtone models are expected to show an asymmetric light curve, resembling the fundamental mode morphology. More in general, at the highest and lowest luminosity levels we find that the predicted light curves have a peculiar behavior which runs against the present observational scenario either of the fundamental pulsators (at \log

$L/L_{\odot}=1.81$) or of the first overtone ones (at $\log L/L_{\odot}=1.21$). Such an evidence would thus constrain the luminosity of actual pulsators roughly to $\log L/L_{\odot} = 1.4 \div 1.6$, in qualitative but relevant agreement with theoretical predictions.

As already discussed in BCCM, the pulsational investigation offers the relevant opportunity of deriving theoretical predictions about the behavior of both period and amplitude of the pulsation. The periods appear in fair agreement with the analytical dependence on luminosity, effective temperature, and mass value given for metal-poor stars in BCCM. Specifically, for each given value of effective temperature and luminosity, the periods for first overtone pulsators do not show relevant differences ($\delta \log P \approx 0.01$), whereas those of fundamental pulsators appear systematically larger than analytical predictions by an amount of $\delta \log P \approx 0.03$.

Figure 4 shows the dependence of the bolometric amplitudes on the effective temperature for the different assumptions on the luminosity level. We find the interesting evidence that around the expected luminosity of HB pulsators (namely, for $\log L/L_{\odot}$ in the range from 1.4 to 1.6) the amplitudes of fundamental pulsators follow, with good accuracy, the amplitude-temperature relation already found for metal-poor variables. More in detail, we find that for each given effective temperature the bolometric amplitude increases very slowly as the luminosity increases. Similarly, first overtone pulsators show the characteristic "bell" shape in the amplitude-temperature plane displayed by metal-poor models even though in this case the maximum amplitude is almost independent of the luminosity level except for the sequence of models at $\log L/L_{\odot}=1.21$.

At the two highest luminosity levels the fundamental amplitudes show an anomalous behavior: for $\log L/L_{\odot}=1.81$ they increase as the effective temperature decreases between 5800 and 5600 K, whereas for $\log L/L_{\odot}=2.0$ they resemble the "bell" shape disclosed by first overtone amplitudes at lower luminosities. This behavior appears related to the peculiar topology of the instability strip at these large luminosities, where only the fundamental mode presents a stable nonlinear limit cycle. In such a case the quoted behaviors can be regarded as evidence that in this region of the instability strip the pulsational amplitudes of fundamental pulsators attain vanishing values close to the instability boundaries. In the region of the instability strip where both fundamental and first overtone present a stable limit cycle, first overtone pulsators attain vanishing amplitudes close to the high temperature edge (FOBE), whereas fundamental pulsators close to the low temperature edge (FRE).

The theoretical period versus bolometric amplitude diagram for selected luminosity levels is reported in Fig. 5. As discussed in BCCM, the topology of data shown in Fig. 5 is easily understood in terms of data in Fig. 4 when the relations connecting periods to

luminosity and to effective temperature are taken into account. The dependence of period on both luminosity and effective temperature removes the "degeneracy" of fundamental amplitudes with stellar luminosity shown in Fig. 4.

3. THE CONNECTION WITH METAL-POOR RR LYRAE VARIABLES

In the previous section we presented theoretical expectations covering the pulsational behavior of "old" ($t > 2.5$, with t in Gyr) He burning stars with solar metallicity. In this way we dealt with a theoretical scenario which should be adequate for a metallicity regarded as a safe upper limit for RR Lyrae stars in the Galactic field. However, field RR Lyrae stars cover a wide range of metallicities, reaching values as low as $[\text{Fe}/\text{H}] \simeq -2.2$ or less, typical of the most metal-poor Galactic globular clusters. The theoretical pulsational scenario for globular cluster RR Lyrae has been already presented in BCCM and it can be safely used also for approaching the problem of field metal-poor pulsators. According to the discussion given in the introduction, in this section we intend to fill the gap between solar metallicity and metal-poor pulsators by investigating the behavior of "mild" metal-rich pulsators with $Z=0.01$ and 0.006 . In this way we will derive a theoretical scenario covering the pulsational properties of RR Lyrae stars over the whole range of metallicity from $Z=0.0001$ to $Z=0.02$.

For the case $Z=0.01$, from Paper I we assume the values $M = 0.58M_{\odot}$ and $Y=0.255$ as suitable evolutionary parameters for old He burning pulsators. Recalling that for similar pulsators one expects a ZAHB luminosity level $\log L/L_{\odot}=1.54$, we explored the pulsational stability for three luminosity levels which should cover the evolutionary behavior of the models, namely $\log L/L_{\odot} = 1.51, 1.57, 1.65$. Concerning $Z=0.006$, we assume the same total mass and helium content with the ZAHB luminosity level $\log L/L_{\odot} = 1.55$. The full results are listed in Tables 4, 5 and 6 in the Appendix, while luminosities and effective temperatures of the computed pulsational boundaries are presented in Table 1.

Figure 6 shows the instability strip with $Z=0.01$ (solid lines) together with the results with $Z=0.006$ (asterisks). In order to point out the dependence of the instability edges on the metal content, in this figure the instability strip for the solar case (dashed lines) is also shown. We find that near the ZAHB luminosity level the boundaries of the instability strip become slightly redder as the metallicity increases, i.e. at fixed helium content, and passing from $Z = 0.006$ to $Z = 0.01$. However, as soon as an increase of both helium and metals is taken into account, the location of the boundaries is only marginally affected and the major difference seems to be connected with the topology of the region between FBE and FORE, the so-called "OR" region.

Figures 7a-b show the atlas of theoretical light curves computed with $Z=0.01$ and the three luminosity levels, while Fig. 8 refers to the sequence of models with $Z=0.006$. Again we find that theoretical light curves appear in good qualitative agreement with the observational scenario. Moreover, we find that the periods of first overtone pulsators follow the analytical relation given in BCCM ($\delta \log P \approx 0.01$), while the periods of fundamental pulsators are systematically larger by $\delta \log P \approx 0.02$ in comparison with the values provided by the quoted relations.

Figure 9 shows the relation between bolometric amplitude and effective temperature for the $Z=0.01$ (solid lines) and $Z=0.006$ (dotted lines) pulsating models. Data plotted in this figure clearly display that the bolometric amplitudes are correlated with effective temperatures, but here with a very slow dependence on the assumed luminosity. Moreover, the same figure shows that for each given temperature the two sequences of models at $Z=0.01$ and 0.006 are characterized by bolometric amplitudes moderately larger if compared with the amplitudes obtained by adopting roughly the same luminosity but a solar metal content (dashed lines). Conclusions in a recent paper by BIM state that, while all physical parameters are constant, the fundamental amplitudes slowly increase with increasing Z . Therefore the opposite behavior found in Fig. 9 could be due to the increase of the helium content connected with the increase of metals. As a matter of fact, the comparison between metal-poor pulsators with $Y=0.30$ (BS) and $Y=0.24$ (BCCM) suggests that at fixed luminosity and effective temperature the bolometric amplitude decreases if helium increases. Moreover, some models computed with $Y=0.31$, $Z=0.02$, $\log L/L_\odot=1.51$ and $0.53M_\odot$ (plotted as asterisks in Fig. 9) actually show smaller amplitudes than similar models constructed by adopting $Y=0.28$.

Theoretical predictions concerning the period-bolometric amplitude relation for the cases $Z=0.01$ (solid lines) and $Z=0.006$ (dotted lines) are finally given in Fig. 10, in comparison with the results (dashed lines) for mild metal-poor pulsators with $Y=0.24$, $Z=0.001$ (BCCM). The models plotted in this figure have been computed by adopting the same mass value and therefore it allows for the analysis of the dependence of bolometric amplitudes on chemical composition. A glance at the curves plotted in this figure show that fundamental amplitudes at the ZAHB luminosity level slightly decrease as the metal and the helium contents increase. On the other hand, first overtone amplitudes do not show a substantial dependence on the chemical composition and the main difference is connected with the change in the period range.

4. RR LYRAE VARIABLES IN THE GALACTIC FIELD

The pulsational theoretical scenario presented in the previous sections can now be connected with the evolutionary constraints presented in Paper I in order to produce theoretical predictions about the expected behavior of metal-rich RR Lyrae pulsators. As a first step, we investigate the well known finding (Preston 1959) for which field metal-rich RR Lyrae stars are characterized by shorter periods when compared with metal-poor variables.

For a discussion of this point, we can rely on both the theoretical results concerning the location in the HR diagram of the blue boundaries for radial instability, derived from pulsational models, and the predictions concerning stellar masses and luminosities at those boundaries derived from evolutionary models. In this framework predictions about the fundamental and first overtone minimum periods can be obtained for the different assumptions on metallicity, as reported in Fig. 11.

The theoretical shortest period for first overtone pulsators is calculated at the intersection between ZAHB (see Table 6 in Paper I) and first overtone blue edge. Whereas, for the minimum fundamental period, we assume that the transition from fundamental to first overtone pulsators occurs close to the fundamental blue edge or to the first overtone red edge. On this basis the region in Fig. 11 located between the FBE and FORE lines represents the "OR" region, i.e. the region of the instability strip where a variable presents a stable limit cycle in both fundamental and first overtone modes. It is worth noting that the theoretical scenario was obtained by assuming, for all metallicities, "old" He burning pulsators. Moreover, for $Z < 0.006$ (solid lines) we adopted $Y_{MS}=0.23$, while for $Z \geq 0.006$ (dashed lines) an enrichment ratio between helium and heavy elements $\Delta Y_{MS}/\Delta Z \approx 3$ was taken into account.

Figure 12 presents the comparison between the theoretical predictions of Fig. 11 for fundamental pulsators and the observational data provided by Blanco (1992, hereinafter VBL92) for field *ab*-type RR Lyrae variables. The metallicities, based on Butler (1975) ΔS index, have been obtained by adopting the relation provided by Suntzeff, Kinman & Kraft (1991), which, in turn, relies on the Zinn & West (1984) metallicity scale. Open circles are RR Lyrae stars with uncertain or unknown blue amplitude, while asterisks refer to variables with unreliable ΔS index. The error-bar refers to the different calibrations of ΔS index as a function of $[\text{Fe}/\text{H}]$ found in the literature (Butler 1975; Suntzeff et al. 1991; VBL92; Jurcsik 1995; Fernley & Barnes 1996).

Figure 12 reveals the interesting evidence that the present pulsational-evolutionary scenario appears in fair agreement with RR Lyrae stars in the Galactic field. As for the metal-poor variables ($[\text{Fe}/\text{H}] < -1.4$), they appear to reproduce the exhaustively discussed

behavior of RR Lyrae variables belonging to Galactic globular clusters, with evidence for the occurrence of the Oosterhoff phenomenon. As a matter of fact, among the most metal-poor stars we recognize the lack of fundamental pulsators in the "OR" region and hence the larger values of the mean period which characterize Oosterhoff type II (OoII) pulsators (see BCCM and references therein). By increasing the metallicity, the "OR" region begins to be filled and the mean period decreases, with the pulsational behavior moving toward typical Oosterhoff type I (OoI) pulsators.

Figure 12 also shows that for pulsators with $[\text{Fe}/\text{H}] > -1.4$ the predicted minimum fundamental period, as attained at the fundamental blue edge, decreases with increasing Z and reaches, in agreement with observations, $\log P = -0.45$ at $Z \approx 0.006$. At larger metallicities the predicted FBE periods appear consistent with the lower envelope of the observed distribution, with the exception of few stars and, in particular, TV Lib. Similar *stragglers* can be understood only assuming a lower age and therefore lower luminosities and smaller periods. This seems almost mandatory to account for the deviant star TV Lib. However, it should be noted that metal-rich variables could be evolved stars originated in the low temperature side of the instability strip and that the corresponding evolutionary tracks could be located at effective temperatures lower than FBE. In such a scenario, the predicted minimum period for fundamental pulsators should be larger than the one attained by the variable at FBE and consequently young metal-rich variables could be hidden in the sequence of "old pulsators".

In order to evaluate the effects of observational errors and/or different metallicity calibrations, in Fig. 13 we present the sample of field stars recently analyzed by Layden (1994, 1995, hereinafter LA95) where $[\text{Fe}/\text{H}]_K$ is based on the strength of the CaII K line. Also for this sample the agreement between theoretical predictions and observed data is satisfactory, as well as for the Kemper (1982, hereinafter KE82) sample of field *c*-type variables (Fig. 14).

A much less satisfactory agreement is found when the comparison deals with pulsational amplitudes. Figure 15 discloses the predicted Bailey diagram (blue amplitude *vs* period) for "old" pulsators at the ZAHB luminosity levels derived for different assumptions about stellar metallicities (for the values of stellar mass and luminosity see Table 6 in Paper I) and with the bolometric amplitudes transformed into blue amplitudes by means of Kurucz (1992) atmosphere models.

The bottom panel shows the predicted behavior with $Z=0.0001$ (dotted line), $Z=0.001$ (dashed line), and $Z=0.006$ (solid line), while the upper panel presents the comparison between the $Z=0.006$ case and the results for $Z=0.01$ (dashed line) and $Z=0.02$ (dotted line). As a whole, we derive a fair independence of the amount of metal over the range

$Z = 0.0001 \div 0.02$, thus excluding the occurrence of different sequences as functions of metallicity. However, it is worth underlining that under the hypothesis of young ($t \approx 1$ Gyr) metal-rich pulsators the predicted period-amplitude sequence would move toward smaller periods, as shown by the dashed-dotted line for the case $Z=0.02$, $\log L/L_\odot=1.41$.

Comparison with observational data is given in Figs 16a-b. The bottom panel of Fig. 16a shows the Layden variables with $[\text{Fe}/\text{H}]_K < -1.4$ together with the predicted behavior of fundamental pulsators with $Z=0.0001$, $M = 0.65M_\odot$, and $\log L/L_\odot=1.61$ and 1.72 . In the upper panel the variables with $[\text{Fe}/\text{H}]_K$ in the range from -0.6 to -1.4 are plotted as open circles, while the solid line shows the predicted behavior of fundamental pulsators with $Z=0.006$, $M = 0.58M_\odot$, and $\log L/L_\odot=1.55$. Figure 16b refers to metal-rich RR Lyrae stars ($[\text{Fe}/\text{H}]_K > -0.6$) in comparison with the predicted behavior of "old" and "young" ZAHB pulsators with $Z=0.02$. As a whole, there is a fair agreement for the metal-poor variables, whereas the behavior of metal-intermediate and metal-rich components seems too discrepant to be accepted. Again, it should be noted that predicted amplitudes for "young" solar metallicity pulsators easily match observations (dashed-dotted line in Fig. 16b). We conclude that either the theory overestimates the amplitude of metal-rich RR Lyrae variables or their behavior in the Bailey diagram reveals the large occurrence of "young pulsators".

5. RR LYRAE VARIABLES IN THE GALACTIC BULGE

The pulsational scenario discussed in the previous sections can be also compared with the pulsational behavior of RR Lyrae stars in the Galactic bulge. Baade (1951) observing in the so-called Baade's Window ² (BW, centered on $l = 1^\circ.0$, $b = -3^\circ.9$) first discovered RR Lyrae stars in the Galactic bulge, using the magnitudes of these variables to estimate the distance to the Galactic center. With the improvement of photometric techniques further variables were discovered in the BW (e.g. Oort & Plaut 1975; Blanco & Blanco 1985; Walker & Mack 1986), causing increasing attention to be devoted to this population of variable stars.

Blanco (1984, hereinafter BBL84) investigated a sample of 77 RR Lyrae variables in the BW suggesting that, in spite of a significant range of metallicity covered by these stars, most of them are relatively more metal-rich than stars in the Galactic globular cluster M3. The range of magnitudes appeared to be quite narrow, with important implications for the controversy about the slope of the RR Lyrae magnitude-metallicity relation.

²It is interesting to note that Baade (1958) called this central region of the Galaxy *van Tulder's pole*.

Figure 17 shows the distribution of periods with metallicity, as derived from ΔS values listed by Walker & Terndrup (1991) and calibration by VBL92 and Suntzeff et al. (1991). It appears in fair agreement with theoretical predictions as well as with the behavior of field RR Lyrae stars with similar metallicities. Moreover, due to the moderately large metal abundance, it appears that the transition between fundamental and first overtone pulsators occurs close to the fundamental blue boundary, with no evidence for the OoII behavior.

Figure 18 shows the period-amplitude diagram for BW *ab*-type RR Lyrae stars (lower panel) in comparison with predictions for "old" (dotted line) and "young" solar metallicity pulsators at $\log L/L_{\odot}=1.41$ (dashed-dotted line). Open circles refer to variables with $[\text{Fe}/\text{H}] \geq -1.0$. Also in this case we find a sort of discrepancy between theory concerning old stars and observations, with the conclusions already discussed at the end of the previous section.

The behavior of first overtone pulsators (Fig. 19) adds to this scenario only the evidence that a metallicity $Z \approx 0.01$ is almost suitable for observations and that a component of OoII variables is not present in the BW, as shown by the lack of the decreasing branch characterizing this type of pulsators (see BCCM). This confirms that the bulge metallicity does not reach the low values which are typical of OoII globular clusters. Moreover, first overtone periods group around $\log P \simeq -0.55$, ($P \simeq 0.3$ days)³, in agreement with the expectation for metal-rich stars given in the previous section. A similar secondary peak in the period distribution of first overtones is also present in some Dwarf Spheroidal Galaxies belonging to the Local Group like Ursa Minor (Nemec, Wehlau, & de Oliveira 1988) and Sculptor (Kaluzny et al. 1995, and references therein) and is further stressed by recent data collected by the OGLE project (Udalski et al. 1994, 1995a,b) for pulsators in BW.

Surveys in bulge windows other than the BW are really important, since interstellar absorption in BW is rather uneven and extreme crowding affects the photometric accuracy. A survey of a new Galactic bulge window ($l = 0.6$, $b = -5.5$), which is remarkably uniform in absorption and not as seriously affected by crowding as the BW, was carried out by Blanco (1992, hereinafter BBL92) who discovered and studied 112 new RR Lyrae variables. According to the quoted author, the comparison between the mean periods of R Rab and R Rc in this new window and those in the BW possibly shows a difference which seems to suggest a metallicity gradient between the two windows.

Data for fundamental pulsators in the new quoted window are reported in the upper

³This peculiarity in the period distribution of RR Lyrae variables in the central region of our Galaxy was originally pointed out by Baade (1958) long before the metal abundances of these objects had been estimated !

panel of Figure 18. As a whole, the pulsational behavior of variables in both windows appears rather homogeneous, not giving clear evidence for the suggested metallicity gradient. Moreover, the comparison with Figs. 16a-b suggests that, at least as far as RR Lyrae stars are concerned, the metal-rich component of the bulge population does not differ from similar stars present in the Galactic field.

6. SUMMARY AND CONCLUSIONS

In a homogeneous theoretical context we developed both evolutionary and pulsational properties of low-mass helium burning stars with chemical compositions typical of field RR Lyrae variables. The evolutionary calculations, based on canonical HB models, have been already discussed in a previous companion paper (Paper I). The pulsation characteristics and the modal stability of hydrodynamical envelope models have been derived by adopting a nonlinear, nonlocal and time-dependent convective approach. For each given chemical composition the sequences of pulsating models were constructed by using both the stellar masses and the luminosity levels predicted by HB evolutionary tracks. On the basis of this thorough theoretical investigation and of the comparison with the presently available observational scenario concerning RR Lyrae variables in the Galactic field and in the Galactic bulge, we can draw the following conclusions:

a. At fixed stellar mass and chemical composition the shape of the light curve of metal-rich pulsators shows a dependence on both effective temperature and luminosity quite similar to that of metal-poor RR Lyrae models constructed by BS and BCCM. Besides some minor differences connected with the appearance of secondary features along the light curves, the instability strip topology of metal-rich pulsators (fundamental and first overtone) is shifted toward luminosities lower than those of metal-poor pulsators.

b. On the basis of few selected models computed by adopting fixed stellar mass, luminosity level, and metal content but different helium contents ($Y=0.28, 0.31$) we find that the bolometric amplitude of fundamental pulsators decreases as helium increases.

c. The comparison in the period-metallicity plane between Galactic field fundamental variables collected by VBL92 and theoretical predictions shows, within the range of observational uncertainties connected with the metallicity calibration of the ΔS index, a satisfactory agreement over a wide metallicity range (2 dex). At the same time there is evidence of the appearance of the Oosterhoff dichotomy among field variables. In fact, in the "OR" region there is substantial lack of metal-poor ($[Fe/H] < -1.7$) fundamental pulsators, which, in turn, implies that this group of pulsators resembles OoII variables,

whereas for higher metal contents field variables follow the pulsation characteristics of OoI variables. The agreement between theory and observations is still satisfactory if we take into account the sample of field fundamental variables recently provided by Layden (1995). The outcome is similar for the sample of first overtone field variables collected by Kemper (1982).

d. The results obtained from the comparison in the Bailey plane -blue amplitude versus period- between theoretical predictions and observations are much less straightforward. As a matter of fact, the B amplitudes of metal-poor ($[\text{Fe}/\text{H}] < -1.4$) pulsators are in fair agreement with theoretical amplitudes obtained by assuming the ZAHB luminosity levels of "old" pulsators. On the contrary, the predicted amplitudes for both metal-intermediate ($-1.4 < [\text{Fe}/\text{H}] < -0.6$) and metal-rich ($[\text{Fe}/\text{H}] > -0.6$) pulsators are systematically larger in comparison with the observed ones. However, in this context it is worth noting that the predicted amplitudes for "young" ($t \approx 1$ Gyr) solar metallicity pulsators provide a satisfactory fit for observational data.

e. The comparison between theoretical predictions and observational data was also extended to the RR Lyrae variables belonging to the Galactic bulge. On the basis of both observational data provided by BBL84 and BBL92 and the theoretical scenario previously outlined, the comparison in the period-metallicity plane is quite satisfactory, whereas the Bailey diagram presents the same discrepancy disclosed by field RR Lyrae variables. The assumption of "young" pulsators could provide, for this sample of variables, a good fit for the pulsational amplitudes of both fundamental and first overtone pulsators.

f. The nonlinear fundamental periods of the present survey are systematically larger ($\delta \log P \approx 0.03$) in comparison with the periods given by the analytical relation suggested by BCCM. This discrepancy clearly suggests that when moving from metal-poor to metal-rich pulsators the nonlinear periods show a nonnegligible dependence on both helium and/or metal contents.

g. As a final point, the pulsational scenario discussed in this paper allows for an approach to the results of the MACHO survey of RR Lyrae in the Large Magellanic Cloud presented by Alcock et al. (1993, 1995, 1996). As for the period-amplitude diagram, we note that the discussion in that paper was partially based on the assumption of different sequences characterizing OoI and OoII fundamental pulsators in the Bailey diagram, an occurrence already challenged both on theoretical (BCCM) and observational (Brocato, Castellani & Ripepi 1996) grounds and confirmed here. As a more relevant point, it has been suggested that the peak in the period frequency distribution of first overtone pulsators located at $\log P \simeq -0.55$ is the possible evidence of second overtone RR Lyrae pulsators. Here we advance the hypothesis that such a peak could be taken as evidence of a metal-rich

population, in agreement with the behavior we already found for both Galactic field and Galactic bulge variables.

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A. NONLINEAR MODEL SUMMARY: PULSATIONAL AMPLITUDES

Although several and thorough investigations have been devoted to the nonlinear systematic properties of RR Lyrae variables in Galactic globular clusters since Christy's seminal paper (1966), we still lack detailed analysis of the pulsation behavior of metal-rich RR Lyrae variables observed in the Galactic field and in the Galactic bulge. With the aim to fill this gap, in this Appendix we report the results of the nonlinear survey of RR Lyrae models we computed. The sequences of envelope models have been constructed by assuming different stellar masses and chemical compositions. Moreover, in order to provide a comprehensive theoretical framework for the pulsation properties inside the instability strip, we adopted a wide range of luminosity levels and effective temperatures. Since we are also interested in the modal behavior of envelope models, the stability analysis was performed for both fundamental and first overtone modes. With the primary purpose of locating the exact instability boundaries of these modes the structure equations have been followed in time until the radial motions approach their limiting amplitude. The reader interested to the methods and the assumptions adopted for constructing the pulsation models is referred to BS, BCCM, and BIM.

The overall stability results of this survey have been already discussed in previous sections. In Tables 2 to 6 are reported the pulsational amplitudes to be compared with actual RR Lyrae observational properties. Similar data but referred to metal-poor pulsators have been already discussed by BCCM. Each table gives -left to right- 1) logarithmic luminosity level; 2) nonlinear period (days); 3) effective temperature (K); 4) fractional radius variation, $\Delta R/R_{ph} = (R^{max} - R^{min})/R_{ph}$ where R_{ph} is the photospheric radius; 5) radial velocity amplitude (km/sec), $\Delta u = u^{max} - u^{min}$; 6) bolometric amplitude (mag), $\Delta M_{bol} = M_{bol}^{max} - M_{bol}^{min}$; amplitude of logarithmic surface gravity, $\Delta \log g = \log g^{max} - \log g^{min}$; 7) static, $g_s = GM/R_{ph}^2$, and 8) effective, $g_{eff} = GM/R_{ph}^2 + du/dt$; 9) surface temperature variation, $\Delta T = T^{max} - T^{min}$ where T is the temperature of the outer boundary; 10) effective temperature variation, $\Delta T_e = T_e^{max} - T_e^{min}$ where T_e is derived from the surface luminosity and radius variations along a full cycle.

The temperature values listed in columns 9 and 10 have been rounded up to the nearest 50 K. In these tables we have reported only models which approach a stable nonlinear limit cycle. The models which, during the integration, experience a mode switch (fundamental toward first overtone or vice versa), or mixed-mode features (two or more radial modes are contemporary excited) are marked by upper letters located close to the value of the effective temperature. Close to the instability boundaries we adopted an effective temperature step of 100 K. Therefore at fixed luminosity level the blue (red) edge of each mode can be easily derived by increasing (decreasing) by 50 K the effective temperature of the first (last) stable model reported in the following tables.

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B. Figure Captions

Fig. 1. The HR diagram location of the instability strip for fundamental and first overtone modes computed at fixed stellar mass ($M/M_{\odot}=0.53$) and solar metallicity (solid lines) as compared with the instability strip for metal-poor stars (dashed lines). The dotted lines show the ZAHB luminosity levels for the labeled metal abundances.

Fig. 2. Theoretical light curves of solar metallicity fundamental pulsators over two consecutive periods. The labels show the luminosity levels and the effective temperatures.

Fig. 3. Same as in Fig. 2, but the light curves are referred to first overtone pulsators.

Fig. 4. Fundamental (solid lines) and first overtone (dashed lines) bolometric amplitudes versus effective temperatures for solar metallicity RR Lyrae variables. The luminosity levels are indicated by different symbols.

Fig. 5. Fundamental (solid lines) and first overtone (dashed lines) bolometric amplitudes versus periods for solar metallicity RR Lyrae variables. The luminosity levels are indicated by different symbols.

Fig. 6. Comparison between the instability strips of models with $M = 0.58M_{\odot}$, $Y=0.255$, $Z=0.01$ (solid lines) and the solar metallicity models (dashed lines). The asterisks mark the instability boundaries for the sequence of models with $M = 0.58M_{\odot}$, $Y=0.255$, and $Z=0.006$. The dotted line shows the ZAHB luminosity level for $Z=0.01$ models.

Fig. 7a. Theoretical light curves over two consecutive periods for fundamental pulsators with $M = 0.58M_{\odot}$, $Y=0.255$, and $Z=0.01$. The labels show the luminosity levels and the effective temperatures.

Fig. 7b. Same as in Fig. 7a, but the light curves are referred to first overtone pulsators.

Fig. 8. Same as in Fig. 7a, but the light curves are referred to fundamental and first overtone pulsators with $M = 0.58M_{\odot}$, $Y=0.255$, and $Z=0.006$.

Fig. 9. The relation between bolometric amplitude and effective temperature for fundamental and first overtone pulsators with $M = 0.58M_{\odot}$, $Y=0.255$, and $Z=0.01$ (solid lines) as compared with similar results but for different chemical compositions, namely

$Y=0.28$, $Z=0.02$ (dashed lines) and $Y=0.255$, $Z=0.006$ (dotted lines). The asterisks mark the bolometric amplitudes of fundamental pulsators computed by adopting $Y=0.31$ and $Z=0.02$.

Fig. 10. The relation between bolometric amplitudes and periods at fixed stellar mass $M = 0.58M_{\odot}$ and different chemical compositions. The labels show the adopted chemical compositions and the luminosity levels.

Fig. 11. Theoretical expectations about the minimum period as a function of stellar metallicity. The minimum fundamental period is evaluated assuming that the transition between fundamental and first overtone pulsators occurs either at the fundamental blue edge (FBE) or at the first overtone red edge (FORE). The minimum first overtone period is evaluated at the first overtone blue edge (FOBE). Solid lines refer to $Y_{MS}=0.23$, whereas dashed lines refer to an original He which increases with increasing Z (see text for further details).

Fig. 12. Comparison between predicted minimum fundamental periods and *ab*-type RR Lyrae stars in the Galactic field. Observational data from VBL92. Open circles refer to RR Lyrae with uncertain or unknown blue amplitudes, while asterisks display variables with unreliable ΔS index.

Fig. 13. Same as in Fig. 12 but with observational data from Layden (1994) and LA95.

Fig. 14. Comparison between predicted minimum first overtone periods and *c*-type RR Lyrae stars in the Galactic field. Observational data from KE82.

Fig. 15. (*Bottom:*) The predicted Bailey diagram for "old" ZAHB pulsators with $Z=0.0001$ (dotted line), $Z=0.001$ (dashed line), and $Z=0.006$ (solid line). (*Top:*) The predicted Bailey diagram for "old" ZAHB pulsators with $Z=0.02$ (dotted line), $Z=0.01$ (dashed line), and $Z=0.006$ (solid line). The dotted-dashed line refers to "young" pulsators with $Z=0.02$.

Fig. 16a. Comparison of predicted blue amplitudes and periods with observational data for selected field *ab*-type RR Lyrae variables from Layden (1994) and LA95. Full circles show variables with $[\text{Fe}/\text{H}] < -1.4$, while open circles refer to variables with $[\text{Fe}/\text{H}]$ in the range from -1.4 to -0.6. The two dashed lines refer to ZAHB pulsators with $Z=0.001$ and two different choices of luminosity ($\log L/L_{\odot}=1.61, 1.72$), while the solid line refers to

ZAHB pulsators with $Z=0.006$.

Fig. 16b. Same as in Fig. 16a but for field variables with $[\text{Fe}/\text{H}] > -0.6$ and with the predicted blue amplitudes and periods referred to "old" (dotted line) and "young" (dashed-dotted line) ZAHB pulsators with $Z=0.02$.

Fig. 17. Same as in Fig. 12 and Fig. 14, but for *ab*-type and *c*-type RR Lyrae stars in the Galactic bulge as given by BBL84.

Fig. 18. Same as in Fig. 16b, but for fundamental pulsators in the BW (bottom panel: BBL84) with $[\text{Fe}/\text{H}] \geq -1.0$ (open circles) and $[\text{Fe}/\text{H}] < -1.0$ (full circles) and in the new Galactic bulge window discussed in the text (top panel: BBL92).

Fig. 19. The location in the Bailey diagram of *c*-type RR Lyrae variables in the BW (BBL84) in comparison with predicted blue amplitudes and periods of first overtone pulsators with $Z=0.01$ and $\log L/L_{\odot} = 1.51$ (dashed line), 1.57 (dashed-dotted line), and 1.65 (dotted line).

TABLE 2
NONLINEAR FUNDAMENTAL SURVEY FOR $M = 0.53M_{\odot}$, $Y = 0.28$, $Z = 0.02$

$\log L/L_{\odot}$	T_e^a	P^b	$\Delta R/R_{ph}^c$	Δu^d	ΔM_{bol}^e	$\Delta \log g_s^f$	$\Delta \log g_{eff}^g$	ΔT^h	ΔT_e^i
2.00	6600	1.2094	.097	36.47	.676	.08	.51	850	1050
	6400	1.3500	.158	68.75	1.173	.14	1.10	1350	1650
	5900	1.8272	.269	90.62	1.246	.23	1.40	1450	1800
	5600 ^j	2.1742	.276	76.47	.900	.25	1.37	1200	1450
	5500	2.3219	.277	81.72	.857	.23	1.30	1150	1400
	5300	2.6074	.247	61.27	.723	.22	1.26	1000	1250
	5200	2.7658	.238	56.68	.665	.22	1.20	950	1150
	5000	3.1044	.156	32.21	.380	.14	.59	550	700
1.81	6700	.7983	.129	71.88	1.074	.11	.82	1350	1700
	6600	.8407	.141	81.65	1.133	.12	.92	1450	1750
	5800	1.3132	.107	40.99	.416	.09	.56	500	600
	5700	1.3966	.147	54.97	.550	.13	.77	700	850
	5500	1.5729	.179	62.70	.698	.16	.95	900	1100
	5400	1.6650	.167	51.97	.588	.15	.76	800	950
1.61	6900	.4936	.141	93.45	1.292	.12	1.15	1850	2250
	6800	.5182	.148	97.21	1.265	.13	1.08	1750	2150
	6700	.5444	.149	94.96	1.181	.13	1.08	1550	1900
	6600	.5724	.150	90.17	1.070	.13	1.15	1350	1650
	6200	.7051	.130	69.51	.618	.11	.83	700	900
	5900	.8316	.082	35.18	.315	.07	.44	450	550
	5800	.8796	.061	25.82	.204	.05	.29	300	400
	5700	.9310	.028	10.21	.082	.02	.12	150	150
1.51	6800	.4284	.154	101.11	1.264	.13	1.22	1800	2200
	6700	.4500	.155	97.90	1.142	.14	1.22	1550	1900
	6600	.4729	.154	91.82	1.012	.14	1.22	1300	1600
	6500	.4973	.151	85.85	.872	.13	1.20	1050	1300
	6400	.5235	.146	80.06	.745	.13	1.14	850	1050
	6000	.6475	.098	46.87	.377	.09	.51	500	650
	5900	.6842	.070	29.39	.239	.06	.35	350	400
1.41	7000 ^k	.2298	.065	59.15	.541	.06	.75	750	950
	6900	.3378	.155	110.70	1.378	.14	1.35	2000	2500
	6700	.3722	.159	97.00	1.103	.14	1.30	1900	1500
	6300	.4557	.139	70.53	.551	.12	1.05	800	1000
	6100	.5065	.109	51.59	.387	.10	.66	550	700
	6000	.5346	.075	32.92	.232	.06	.37	400	450
1.21	6900 ^k	.1662	.089	84.03	.562	.08	1.11	850	1050
	6800	.2431	.153	103.93	1.096	.13	1.16	1450	1800
	6700	.2552	.155	104.02	.938	.14	1.10	1200	1450
	6400	.2963	.135	81.47	.566	.12	.97	850	1000
	6200	.3285	.032	17.17	.091	.03	.16	150	200

^a Effective temperature (K). ^b Period (days). ^c Fractional radius variation. ^d Radial velocity amplitude (Km/s). ^e Bolometric amplitude (mag.). ^f Amplitude of logarithmic static gravity. ^g Amplitude of logarithmic effective gravity. ^h Surface temperature variation. ⁱ Effective temperature variation. ^j Mixed-mode pulsator. The amplitudes referred to this model are the average values over the last twenty cycles. ^k Model which shows a mode switch from fundamental toward first overtone.

TABLE 3

NONLINEAR FIRST OVERTONE SURVEY FOR $M = 0.53M_{\odot}$, $Y = 0.28$, $Z = 0.02$

$\log L/L_{\odot}$	T_e^a	P^b	$\Delta R/R_{ph}^c$	Δu^d	ΔM_{bol}^e	$\Delta \log g_s^f$	$\Delta \log g_{eff}^g$	ΔT^h	ΔT_e^i
1.81	6700 ^j	.5694	.054	32.77	.354	.05	.42	550	650
	6500 ^k	.8867	.150	85.44	1.119	.13	.94	1400	1700
	6400 ^k	.9357	.152	84.42	1.032	.13	.94	1200	1500
	6200 ^k	1.0417	.146	72.31	.776	.13	.88	850	1050
1.61	7000	.3352	.040	27.55	.333	.03	.35	500	600
	6800	.3701	.058	45.75	.435	.05	.55	550	700
	6700 ^j	.3892	.063	48.75	.463	.06	.59	600	700
	6500	.4318	.080	56.87	.506	.07	.70	650	800
	6400	.4554	.100	65.37	.463	.08	.80	600	750
	6300	.4795	.097	59.14	.430	.08	.82	600	750
	6200	.5045	.078	42.48	.308	.07	.62	450	550
	6100 ^k	.7443	.120	60.13	.521	.10	.66	650	800
	6000 ^k	.7863	.104	46.38	.433	.09	.58	600	700
1.51	7100	.2643	.044	34.42	.415	.04	.44	600	750
	7000	.2772	.052	39.78	.454	.04	.54	650	800
	6900	.2909	.056	42.90	.450	.05	.58	650	800
	6800	.3056	.058	44.95	.426	.05	.58	600	700
	6750	.3133	.059	45.60	.406	.05	.57	550	700
1.41	7300	.2000	.028	23.30	.296	.02	.26	450	550
	7200	.2093	.052	48.88	.516	.05	.62	750	950
	6800	.2530	.068	54.12	.466	.06	.77	650	750
	6600	.2793	.053	39.66	.276	.05	.51	400	500
	6500 ^k	.4113	.153	85.05	.771	.13	1.25	1000	1200
	6400 ^k	.4328	.147	78.48	.634	.13	1.16	900	1100
	6300 ^k	.4557	.139	70.53	.551	.12	1.05	800	1000
1.21	7500	.1274	.069	76.46	.889	.06	.89	1450	1750
	7400	.1330	.082	86.38	.985	.07	1.08	1600	1950
	7200	.1450	.092	90.89	.911	.08	1.22	1450	1800
	7000	.1587	.092	88.45	.689	.08	1.19	1000	1250
	6700	.1828	.071	59.00	.403	.06	.80	600	750
	6600 ^k	.2680	.153	101.25	.757	.13	1.12	1000	1200
	6500 ^k	.2817	.147	93.14	.662	.13	1.06	950	1150

^a Effective temperature (K). ^b Period (days). ^c Fractional radius variation. ^d Radial velocity amplitude (Km/s). ^e Bolometric amplitude (mag.). ^f Amplitude of logarithmic static gravity. ^g Amplitude of logarithmic effective gravity. ^h Surface temperature variation. ⁱ Effective temperature variation. ^j Mixed-mode pulsator. The amplitudes referred to this model are the average values over the last twenty cycles. ^k Model which shows a mode switch from first overtone toward fundamental.

TABLE 4

NONLINEAR FUNDAMENTAL SURVEY FOR $M = 0.58M_{\odot}$, $Y = 0.255$, $Z = 0.01$

$\log L/L_{\odot}$	T_e^a	P^b	$\Delta R/R_{ph}^c$	Δu^d	ΔM_{bol}^e	$\Delta \log g_s^f$	$\Delta \log g_{eff}^g$	ΔT^h	ΔT_e^i
1.65	6900	.4924	.135	96.16	1.441	.12	1.77	2050	2550
	6800	.5171	.151	105.85	1.523	.13	1.86	2150	2650
	6200	.7051	.152	83.69	.735	.13	1.23	800	1000
	5900	.8353	.118	63.82	.402	.10	.67	600	700
	5700	.9333	.056	23.52	.179	.05	.26	250	350
1.57	7000	.4038	.120	91.86	1.382	.11	1.48	2000	2450
	6900	.4237	.145	109.77	1.584	.13	1.88	2300	2850
	6800	.4448	.159	113.04	1.576	.14	1.81	2250	2750
	6700	.4671	.167	111.97	1.492	.14	1.75	2000	2550
	6600	.4911	.171	107.71	1.346	.15	1.53	1800	2200
	6100	.6388	.145	76.00	.553	.13	1.09	750	900
	5900	.7135	.110	56.74	.393	.10	.67	550	650
	5800	.7568	.080	40.29	.244	.07	.39	350	450
1.51	7000	.3602	.133	106.46	1.570	.12	1.85	2300	2850
	6900	.3738	.151	114.92	1.629	.13	1.90	2400	2950
	6500	.4604	.173	101.18	1.149	.15	1.50	1500	1850
	6100	.5684	.145	74.06	.557	.13	1.12	750	950
	5900	.6345	.102	49.71	.341	.09	.62	500	600

^a Effective temperature (K). ^b Period (days). ^c Fractional radius variation. ^d Radial velocity amplitude (Km/s). ^e Bolometric amplitude (mag.). ^f Amplitude of logarithmic static gravity. ^g Amplitude of logarithmic effective gravity. ^h Surface temperature variation. ⁱ Effective temperature variation.

TABLE 5

NONLINEAR FIRST OVERTONE SURVEY FOR $M = 0.58M_{\odot}$, $Y = 0.255$, $Z = 0.01$

$\log L/L_{\odot}$	T_e^a	P^b	$\Delta R/R_{ph}^c$	Δu^d	ΔM_{bol}^e	$\Delta \log g_s^f$	$\Delta \log g_{eff}^g$	ΔT^h	ΔT_e^i
1.65	7000	.3394	.018	12.81	.179	.02	.17	250	350
	6900	.3565	.054	40.29	.477	.05	.56	650	850
	6600	.4146	.070	59.86	.525	.06	.67	650	800
	6500 ^j	.6019	.165	100.46	1.186	.14	1.38	1550	1950
1.57	7100	.2788	.024	18.79	.254	.02	.23	350	450
	6900	.3068	.056	48.73	.531	.05	.69	750	900
	6700	.3386	.068	53.16	.509	.06	.74	650	750
	6600	.3561	.068	56.39	.461	.06	.69	600	700
	6500 ^j	.5168	.170	100.08	1.160	.15	1.44	1500	1850
1.51	7100	.2490	.046	45.88	.492	.04	.58	750	900
	6900	.2738	.064	63.02	.633	.06	.79	900	1100
	6700	.3018	.073	56.01	.521	.06	.82	700	850
	6600	.3173	.072	55.19	.474	.06	.78	600	750

^a Effective temperature (K). ^b Period (days). ^c Fractional radius variation. ^d Radial velocity amplitude (Km/s). ^e Bolometric amplitude (mag.). ^f Amplitude of logarithmic static gravity. ^g Amplitude of logarithmic effective gravity. ^h Surface temperature variation. ⁱ Effective temperature variation. ^j Model which shows a mode switch from first overtone toward fundamental.

TABLE 6
NONLINEAR SURVEY FOR $M = 0.58M_{\odot}$, $\text{LOG } L/L_{\odot}=1.55$, $Y = 0.255$, $Z = 0.006$

T_e^a	P^b	$\Delta R/R_{ph}^c$	Δu^d	ΔM_{bol}^e	$\Delta \log g_s^f$	$\Delta \log g_{eff}^g$	ΔT^h	ΔT_e^i
Fundamental								
7100	.3633	.129	102.40	1.559	.11	1.89	2350	2900
7000	.3810	.150	113.02	1.682	.13	2.05	2550	3100
6900	.3996	.165	119.84	1.661	.14	2.00	2500	3050
6800	.4194	.177	120.82	1.581	.15	1.85	2350	2900
6500	.4879	.184	106.63	1.116	.16	1.53	1450	1750
6100	.6041	.145	71.56	.517	.13	1.12	700	900
5900	.6749	.085	39.37	.243	.08	.48	400	450
First Overtone								
7200	.2550	.034	29.62	.376	.03	.34	550	700
7100	.2671	.053	56.60	.585	.05	.72	850	1050
6700	.3237	.078	60.01	.515	.07	.89	700	850
6600 ^j	.4631	.180	114.28	1.303	.16	1.58	1800	2200

^a Effective temperature (K). ^b Period (days). ^c Fractional radius variation. ^d Radial velocity amplitude (Km/s). ^e Bolometric amplitude (mag.). ^f Amplitude of logarithmic static gravity. ^g Amplitude of logarithmic effective gravity. ^h Surface temperature variation. ⁱ Effective temperature variation. ^j Model which shows a mode switch from first overtone toward fundamental.